"Quantum optics"

WS 2016-2017 Worksheet 4

This worksheet is devoted to a study of the density operator of two-level systems. The paper is about Wheeler's delayed choice experiment.

Exercise 8: Pair of two-level atoms

Consider two spin-1/2 particles (expressing two two-level atoms). We denote the individual basis states as $|\uparrow\rangle$ and $|\downarrow\rangle$.

1) What are the four basis states for the composite system?
2) There is one state with spin 0 (singlet) and three states with spin 1 (triplet). Calculate the corresponding density operator. Evaluate the reduced density operator of the first spin, obtained by taking the trace over the second spin.
3) Evaluate the purity of each of the above states (with respect to the reduced system). Discuss the physical meaning of the results.

Exercise 9: Density operator of a two-level atom

The simplest and most important model of an atom consists of two discrete and orthogonal states $|b\rangle$ and $|a\rangle$. A complete set of operators used to describe two-level atoms are the four Pauli operators: $\mathbb{1} = |a\rangle\langle a| + |b\rangle\langle b|$, $\sigma_3 = |a\rangle\langle a| - |b\rangle\langle b|$, $\sigma_+ = |a\rangle\langle b|$ and $\sigma_- = |b\rangle\langle a|$ (instead of σ_+ and σ_- the operators $\sigma_1 = \sigma_- + \sigma_+$ and $\sigma_2 = i(\sigma_- - \sigma_+)$ are also used).

1) Write the matrix representation of these 6 operators in the basis $|b\rangle$, $|a\rangle$.
2) The density operator of a general state is given by
$$\rho = \rho_{aa}|a\rangle\langle a| + \rho_{ab}|a\rangle\langle b| + \rho_{ba}|b\rangle\langle a| + \rho_{bb}|b\rangle\langle b| \ .$$
Use 1) to show that $\rho = \frac{1}{2}(\mathbb{1} + u\sigma_1 + v\sigma_2 + w\sigma_3)$ and determine the coefficients u, v and w in terms of the matrix elements of ρ.
3) Show that $\rho_{aa} + \rho_{bb} = 1$ and that $\rho_{aa}\rho_{bb} = \rho_{ab}\rho_{ba}$ for pure states. Show that in general $\rho_{aa}\rho_{bb} \geq \rho_{ab}\rho_{ba}$.

Let us now heuristically derive the equation governing the time evolution of ρ for an atom undergoing spontaneous emission.

4) The excited state probability ρ_{aa} decays at the Einstein rate $2A$. Use the results of 3) to show that $\rho_{aa} \gtrsim |\rho_{ab}|^2$ assuming that $\rho_{aa}^2 \ll \rho_{aa}$. What does this imply for the decay rate of the matrix element ρ_{ab}?
5) Write down the simplest set of equations for $\dot\rho_{ba}$, $\dot\rho_{ab}$, $\dot\rho_{bb}$ and $\dot\rho_{aa}$ consistent with 4). Show that these equations can be combined to the so-called master equation
$$\dot\rho = A(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-) \ .$$
We shall later see that this is indeed the equation determining the time evolution of ρ.

Exercise 10: Paper-Work

Find the following articles online and answer the following questions for each of them:

- What is the paper about?
- Why is it interesting?
- What is done?
- How is it done?

Experimental Realization of Wheeler's Delayed-Choice Gedanken Experiment
Vincent Jacques, E Wu, Frédéric Grosshans, François Treussart, Philippe Grangier, Alain Aspect, Jean-François Roch
Science **315**, 966 (2007)