"Quantum optics"

WS 2016-2017 Worksheet 1

The aim of this worksheet is to analyze Einstein's pre-quantum theory of absorption and emission of light by atoms, and discuss double-slit experiments with single electrons and photons.

Exercise 1: Einstein's theory of atom-radiation interaction

In 1917, Einstein formulated a phenomenological theory for spontaneous and stimulated emission and absorption.
Assume a closed cavity with N identical atoms with two relevant energy levels, E_a and E_b, quasi-resonant with the thermal radiation produced by the cavity walls at temperature T: $\hbar\omega = E_a - E_b$, where ω is the frequency of the photon. The average energy density of the thermal radiation is $u_T(\omega)$.
Let A be the probability per unit time to spontaneously decay from level a to b, emitting a photon of energy $\hbar\omega$. On the other hand, if the atom is in state b, there will be a probability per unit time for absorption proportional to the energy present in the cavity; that is the absorption rate is $Bu_T(\omega)$.

1) Denoting by N_a and N_b the populations of the two levels ($N_a + N_b = N$), write the rate equations for $\dot{N}_a$ and $\dot{N}_b$, where "$\cdot$" is the time derivative.

2) Consider thermal equilibrium and assume that levels a and b are Boltzmann distributed. Determine $u_T(\omega)$.

3) Compare $u_T(\omega)$ with Planck's black body energy distribution and show that the latter is not recovered.

4) Einstein introduced a third process, called stimulated emission, with a rate given by $Bu_T(\omega)$. Write down the modified rate equation and show that now Planck's distribution is recovered. Determine A and B.

5) Solve the rate the rate equation from 4) with $N_a(0) = 0$. What is the lifetime of the upper level in the absence of thermal radiation?

6) We next assume that there is an additional external source of energy, e.g. a light beam crossing the cavity (like in a laser). The average density can then be written as $u(\omega) = u_T(\omega) + u_E(\omega)$, where E denotes the external energy source. Discuss the steady state properties of the solution of the rate equation. What is the maximum possible occupation of the upper level?

Please turn over!

Exercise 2: Paper-Work

Find the following articles online and answer the following questions for each of them:

- What is the paper about?
- Why is it interesting?
- What is done?
- How is it done?

Demonstration of single electron buildup of an interference pattern
A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, and H. Ezawa,
American Journal of Physics **57**, 117 (1989)

The wave-particle duality of light: A demonstration experiment
T. L. Dimitrova and A. Weis
American Journal of Physics **76**, 137 (2008)