"Quantum optics"

WS 2016-2017 Worksheet 3

In this worksheet, we use perturbation theory to microscopically derive the Einstein A and B coefficients and study the density operator of a thermal harmonic oscillator. Moreover, the paper is concerned with an experimental test of the linearity of quantum mechanics and the observation of the Aharonov-Bohm effect.

Exercise 5: Semiclassical evaluation of Einstein's A and B coefficients

We consider a two-level atom (with energies E_a and E_b: $\hbar\omega_{ab} = E_a - E_b$) interacting resonantly with a classical electric field $\vec{E}(t) = E_0\vec{n}\cos(\omega t)$. The interaction Hamiltonian is given by $H^1(t) = -\vec{d}\cdot\vec{E}(t)$, where $\vec{d} = -q\vec{r}$ is the electric dipole moment of the atom. Assume that the atom is initially in the lower state b.

1) Use Fermi's golden rule to obtain the transition probability $P_{b\to a}$.

2) To obtain the B coefficient, we need to average the transition matrix element $|d_{ba}|^2$ over all possible orientations of $\vec{r}$ in 3 dimensions. For a propagation along the z-direction, evaluate the average of $(\vec{n}\cdot\vec{r})^2$ over all possible polarization directions. Average the resulting expression over all propagation directions using spherical coordinates.

3) For a broad-band radiation, the above expression has to be further averaged over the energy density $U(\omega)$ of the radiation. Use the fact that the energy density of the electromagnetic field per unit volume in the frequency range between ω and $\omega + d\omega$ is $U(\omega)d\omega = \frac{1}{2}\epsilon_0 E_0^2$ and assume that $U(\omega)$ varies slowly with ω.

4) Determine A and B using the results of Exercise 1.

Exercise 6: Density operator of a thermal harmonic oscillator

Consider a quantum-mechanical harmonic oscillator coupled to a heat bath at temperature T.

1) Write down the probability to occupy energy eigenstate $|n\rangle$ and the corresponding density operator. Determine the normalization factor (the partition function).

2) Evaluate the mean energy $\langle\hat{H}\rangle$ and determine the average number of quanta $n_{\text{th}} = \langle\hat{n}\rangle$.

3) Express the density operator as a function of n_{th} and compute $(\Delta n_{\text{th}})^2 = \langle\hat{n}^2\rangle - n_{\text{th}}^2$.

Exercise 7: Paper-Work

Find the following articles online and answer the following questions for each of them:

- What is the paper about?
- Why is it interesting?

- What is done?
- How is it done?

Test of the linearity of quantum mechanics by RF spectroscopy of the $^{9}Be^{+}$ *ground state*
J. J. Bollinger, D. J. Heinzen, Wayne M. Itano, S. L. Gilbert, and D. J. Wineland
Phys. Rev. Lett. **63**, 1031 (1989).

Evidence for Aharonov-Bohm effect with magnetic field completely shielded from electron wave
A. Tonomura, N. Osakabe, T. Matsuda, T. Kawasaki, J. Endo, S. Yano, and H. Yamada,
Phys. Rev. Lett. **56**, 792 (1986).