Friedrich-Alexander Universität Erlangen-Nürnberg

WS 2012/2013

Institut für Theoretische Physik II
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13. Übungsblatt Many-body physics with ultra-cold atomic gases

22.01.2013
Besprechung: Thursday, 06.02.2013

13.1: T_c for Bosons at $U = 0$

Compute T_c for free Bosons that are defined on a cupic lattice:

$$H = -J \sum_{\langle ij \rangle} a_i^\dagger a_j \, .$$

13.2: Ground state wave-function at $U = 0$

Argue that the ground state of the Bose-Hubbard model at $U = 0$ can be described as a product of local coherent states in the limit of large particle numbers.

13.3: Bose glass phase in the Bose-Hubbard model

In this exercise we want to get familiar with another phase that can be realized in the Bose–Hubbard model by adding disorder. To answer these questions, you may read these papers: Phys. Rev. B 40, 546 (1989); Phys. Rev. B 80, 214519 (2009) and Phys. Rev. Lett. 103 140402 (2009).

(a) What is a Bose glass? How does on need to change the Bose-Hubbard model to realize this phase (i.e., which particular form of disorder is considered in these papers)?

(b) How is it different from a Mott insulator, how is it different from the superfluid phase?

(c) How does the phase diagram change?

(d) What experimental methods are mentioned in the papers that can be used to realize disorder?

13.4: Local density approximaton I

We consider a one-component Bose gas on a 1D lattice with N particles. Assume that the equation of state $n = n(\mu)$ is known:

$$n = \begin{cases} 0 & \text{for} \quad \mu < 0 \\ 2\tanh(\mu) & \text{for} \quad 0 < \mu \end{cases} \tag{1}$$

Now we add a trapping potential $H_{\text{trap}} = V_0 \sum_i n_i (i - i_0)^2$.

a. Sketch the shell structures that in principle can occur as a function of N and V_0.

b. Given $V_0 = 0.01$ and $N = 20$, calculate the density profile using the local density approximation, i.e., $\langle n_i \rangle = f(\mu_i)$ with $\mu_i = \mu_0 - V_0(i - i_0)^2$.

c. Now redo the calculation for $N = 200$ and $N = 100$. How does one need to choose V_0 in order for the desnity profiles to be identical when plotting them versus i/ξ with $\xi = \sqrt{V_0}^{-1}$?

13.5: Local density approximaton II

We consider a one-component Bose gas on a 1D lattice with N particles. Assume that the equation of state $n = n(\mu)$ is known:

$$n = \begin{cases} 0 & & \mu < 0 \\ \mu^2 & & 0 < \mu < 1 \\ 1 & \text{for} & 1 < \mu < 3/2 \\ \sqrt{\mu - 3/2} + 1 & & \mu > 3/2 \end{cases} \tag{2}$$

Now we add a trapping potential $H_{\text{trap}} = V_0 \sum_i n_i (i - i_0)^2$.

a. Sketch the shell structures that in principle can occur as a function of N and V_0.

b. Given $V_0 = 0.01$ and $N = 100$, calculate the density profile using the local density approximation, i.e., $\langle n_i \rangle = f(\mu_i)$ with $\mu_i = \mu_0 - V_0(i - i_0)^2$.