Friedrich-Alexander Universität Erlangen-Nürnberg

WS 2012/2013

Institut für Theoretische Physik II
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10. Übungsblatt Many-body physics with ultra-cold atomic gases

19.12.2012: Frohe Weihnachten!
Besprechung: Dienstag, 8.01.2013

10.1: Thomas-Fermi approximation

Compute the chemical potential for a Bose gas in a harmonic trap as a function of particle number N using the Thomas-Fermi approximation. From this result, derive the ratio $E_{\text{int}}/E_{\text{trap}}$.

10.2: Healing-length

Verify that

$$\psi(x) = \psi_0 \tanh(x/(\sqrt{2}\xi))$$

is a solution of the 1D Gross-Pitaevskii equation under the boundary conditions discussed in the lecture in 6.2.d.

10.3: Current density

Starting from the time-dependent Gross-Pitaevskii equation, derive the expression for the current density from the equation of continuity.

10.4: Bose-Einstein condensation in experiments

Read the following publications: (i) Anderson et al., Science **269**, 198 (1995), (ii) Davis et al., Phys. Rev. Lett. **75**, 3969 (1995), and (iii) Andrews et al., Science **273**, 84 (1996), and (iv) Onofrio et al., Phys. Rev. Lett. **85**, 2228 (2000). Answer the following questions:

(a) Which atomic species were used to create BECs in (i) and (ii)? Describe the typical experimental sequence to reach quantum degeneracy and BEC in a dilute atomic vapor.

(b) Describe the experimental techniques used in (i) and (ii) to avoid Majorana losses.

(c) List the experimental evidences that were used to prove the formation of a BEC in (i) and (ii).

(d) Explain the relation between the rf-frequency ν_{evap} and temperature T.

(e) How do ^{85}Rb and ^{87}Rb differ from each other?

(f) What are the effects of interactions in the two experiments (i) and (ii), according to their discussion?

(g) Explain the notion of a critical particle number N_c for BEC that is mentioned in (ii). What determines this critical density?

(h) In (ii), the density distribution of condensate fraction $n_0(\boldsymbol{r})$ is argued to be related to the trapping potential $V_{\text{trap}}(\boldsymbol{r})$ via

$$n_0(\boldsymbol{r}) = n_0(\boldsymbol{r} = 0) - V_{\text{trap}}(\boldsymbol{r})/U_0$$

where $U_0 = 4\pi\hbar^2 a/m$ and a is the scattering length. Explain this expression.

(i) In what sense is the measurement technique presented in (iii) non-destructive? How does the measurement affect the state of the system?

(j) In (iv), the sound velocity c_s of the gas at its center is determined from $c_s = 2\pi\nu_z R_z/\sqrt{2}$, where R_z is the Thomas-Fermi radius. Derive this relation.

(k) In (iv), the pressure is equated to $P = n(\boldsymbol{r})\mu(\boldsymbol{r})/2$. Derive this expression.