"Quantum optics"

WS 2016-2017 Worksheet 2

This worksheet is devoted to a study of a simplified laser system based on the Einstein model of exercise 1 and on two experimental articles on the particle-wave duality.

Exercise 3: Elementary theory of the laser

Let us consider two-level atoms resonant with the radiation field (and neglect for the time being spontaneous emission) as in Exercise 1. We write the rate equation in the form

$$\dot{N}_b = -\dot{N}_a = -WnN_b + WnN_a$$

where n is the number of photons in the cavity and W the transition rate from one level to the other.

We define the population difference $D = N_a - N_b$ that obeys

$$\dot{D} = -2WnD - \frac{1}{T_1}(D - D_0)$$

The phenomenological second term on the right hand side accounts for spontaneous decay and pump action. T_1 is the characteristic lifetime associated with the decay of the population, while $D_0 = D(0)$ is the equilibrium population in the absence of photons. On the other hand, there is also a rate equation for the photons:

$$\dot{n} = WnD - \frac{n}{T_c}$$

where the last term describes the photons coming out of the cavity (T_c: lifetime of photons in the cavity).

1) *Threshold and population inversion.* Assuming initially a low photon number (say 1), amplification of the number of photons will only occur if $\dot{n} > 0$. Discuss the physical meaning of this condition: When is it satisfied? What are the implications for the level populations? How can the threshold be lowered?

2) *Steady state.* Determine and discuss the steady state solutions n_∞ and D_∞ corresponding to $\dot{D} = \dot{n} = 0$.

3) *Linear stability analysis.* Let us write the solutions of the laser rate equations in the form

$$n(t) = n_\infty + \epsilon_n(t) \quad \text{and} \quad D(t) = D_\infty + \epsilon_D(t)$$

where n_∞ and D_∞ are the steady state solutions and ϵ_n and ϵ_D are small deviations from the steady state.

By linearizing the laser equations up to order ϵ, derive equations for $\dot{\epsilon}_n$ and $\dot{\epsilon}_D$. Using the ansatz

$$\begin{pmatrix} \epsilon_n \\ \epsilon_D \end{pmatrix} = e^{\lambda t} \begin{pmatrix} \epsilon_n(0) \\ \epsilon_D(0) \end{pmatrix}$$

discuss the stability ($\lambda < 0$) of the solutions for the steady state solutions found in 2). Plot a stability diagram for n_∞ as a function of D_0.

Exercise 4: Paper-Work

Find the following articles online and answer the following questions for each of them:

- What is the paper about?
- Why is it interesting?
- What is done?
- How is it done?

Wave–particle duality of C_{60} molecules
Markus Arndt, Olaf Nairz, Julian Vos-Andreae, Claudia Keller, Gerbrand van der Zouw and Anton Zeilinger
Nature **401**, 680 (1999)

Observation of the Kapitza–Dirac effect
Daniel L. Freimund, Kayvan Aflatooni and Herman Batelaan
Nature **413**, 142 (2001)