Friedrich-Alexander Universität Erlangen-Nürnberg

WS 2012/2013

Institut für Theoretische Physik II
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Sprechstunde: Do. 11-12 Uhr, Raum 02.782.
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11. Übungsblatt Many-body physics with ultra-cold atomic gases

8.01.2013
Besprechung: Mittwoch, 16.01.2013, 8:30 Uhr s.t.

11.1: Phase fluctuations

(a) Consider a 1D (classical) harmonic oscillator

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 \,.$$

Show that

$$\langle e^{ix} \rangle = e^{-\langle x^2 \rangle/2} \,.$$

Use that the moments $\langle x^m \rangle$ follow a Gaussian distribution:

$$\langle x^m \rangle = \begin{cases} (n-1)(n-3)\cdot \dots \cdot 3 \cdot 1 \, \langle x^2 \rangle^{m/2} & \text{for} \quad m \,\text{even} \\ 0 & \qquad\quad m \,\text{odd} \end{cases}$$

(b) Show that

$$\langle \Delta\phi^2 \rangle = 2 \sum_{\boldsymbol{q}, q < q_{max}} \langle |\phi_q|^2 \rangle (1 - \cos(\boldsymbol{q} \cdot \boldsymbol{r})) \,.$$

(c) At which temperature scale T_{2D} does quantum degenerate behavior set in for a non-interacting 2D Bose gas?

(d) Compute the leading dependence on distance r of the OPDM for a 2D Bose gas, including only phase fluctuations. Assume that $r \gg r_m$ where $r_m \sim 1/q_{max}$. Further, exploit that $1 - \cos(\boldsymbol{q} \cdot \boldsymbol{r}) \approx 0$ for $q \ll 1/r$. Interpret your result.

11.2: Excited states of a Tonks-Girardeau gas

The allowed momenta for the eigenstates of the TG gas are:

$$k_j = \frac{2\pi}{L} \left(n_j - \frac{N+1}{2} \right)$$

where n_j is an integer. Each state is thus characterized by a set of N numbers.

(a) Which set of integers yields the ground state of N particles?

(b) An excited state is given by: $\{n_j\} = \{1, 2, \ldots, N-1, N+\lambda\}$ where λ is a positive integer. What is the total momentum of this state and what is the energy of this state with respect to the ground-state energy? Interpret this result, what kind of excitations are these?

(c) Another possible excitation is obtained by removing one integer from the ground state set and by adding $N+1$. What are the total momentum and the energy relative to the ground state energy? Whay type of excitations are these in the (i) fermionic and (ii) bosonic picture?

11.3: Experimental realization of the Tonks-Girardeau gas

Read these original publications on the experimental realization of strongly interacting Bose gases in 1D: (A) Paredes et al., Nature **429**, 277 (2004); (B) Kinoshita et al., Science **305**, 1125 (2004). Answer the following questions:

(a) What is the meaning and definition of the parameter γ discussed in (A)? Which values of γ where realized in (A)?

(b) By means of which experimental methods has the Tonks-Girardeau (TG) been realized in (A)?

(c) What is the dispersion relation of non-interacting bosons in a 1D lattice? What happens as one sends the filling factor ν to zero?

(d) Which properties of the TG gas are identical to non-interacting, spinless fermions? Why?

(e) For a 1D lattice gas of hard-core bosons (i.e., with infinitely strong onsite repulsion), by means of which transformation does the system map to spinless fermions?

(f) What is the experimental evidence reported in (A) for the realization of the TG gas?

(g) Does a non-interacting Bose gas in 1D undergo Bose-Einstein condensation at a nonzero temperature in a harmonic trap?

(h) How is the TG regime realized in the experiment (B)? Which values of γ were reached there?

(i) Why does strengthening the axial confinement decrease γ?

(j) How is the total energy ϵ (the sum of kinetic and potential energy) measured in (B)? Which assumptions underlie this measurement?

(k) Which conditions have to be met for an experiment to realize the 1D regime?