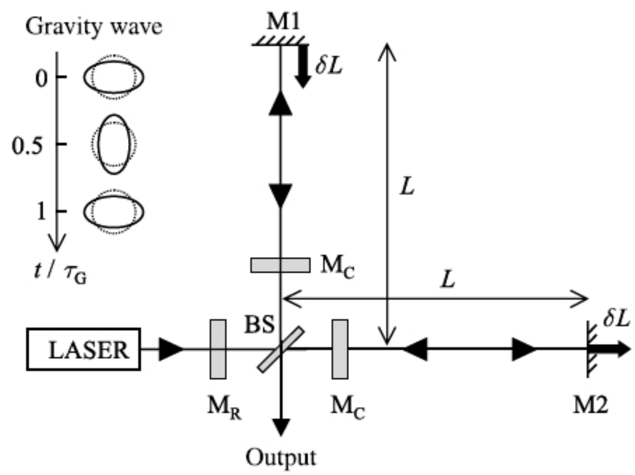
"Quantum optics"

WS 2016-2017 Worksheet 9

This worksheet is devoted to the discussion of the detection of gravitational waves

Exercise 22: Detection of gravitational waves using coherent states



The above figure shows a schematic representation of the LIGO (Light Interferometer Gravitational wave Observatory) experiment, which which has recently detected gravitational waves for the first time. The inset in the upper-left corner shows the effect that a gravitational wave has on a circular object, which is to distort it elliptically. The LIGO experiment attempts to measure this effect by using a Michelson-interferometer type setup. A laser beam is injected into the interferometer via a 50:50 beam splitter (BS). The two interferometer arms of length L are comprised of the the end mirrors $M1$ and $M2$ – both of which are mounted on test masses – and the cavity mirrors M_C, which increase the effective arm length by a factor of 50. The recycling mirror M_R is used to recycle the power in the interferometer by a factor of 60. The effect of a gravitational wave on the interferometer is to displace the end mirrors by δL in opposite directions with respect to the beam splitter. This leads to a periodic oscillation of the observed fringe pattern at the output.
Use the following parameters for the calculations: The laser operates at a wavelength of $\lambda = 1064nm$ with an output power of $5W$. The interferometer arms are $L = 4km$ long.

1) Begin by calculating the average photon flux, defined as the average number of photons per unit time. Calculate the corresponding variance in photon number assuming a coherent state.

2) Use the graphical representation of a coherent state in quadrature space to obtain a relation between the number variance and the phase uncertainty $\Delta\phi$ in the limit of small $\Delta\phi$. ϕ is the average angle with respect to the X_1 axis in quadrature space. Determine the minimal value of $\Delta\phi$.

3) Calculate the phase shift between the two laser beam in the interferometer arms due to the length change δL. What is the smallest observable displacement δL?

4) Evaluate the minimal relative length change (with respect to the original length) that can be detected. How can this sensitivity be improved?

Exercise 23: Paper-Work

Find the following articles online and answer the following questions for each of them:

- What is the paper about?
- Why is it interesting?
- What is done?
- How is it done?

A quantum-enhanced prototype gravitational-wave detector
K. Goda, O. Miyakawa, E. E. Mikhailov, S. Saraf, R. Adhikari, K. McKenzie, R. Ward, S. Vass, A. J. Weinstein and N. Mavalvala, Nature Physics **4**, 472 - 476 (2008)

Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light.
J. Aasi *et al.*, Nature Photonics, **7**, 613 - 619 (2013).

Observation of Gravitational Waves from a Binary Black Hole Merger
B. P. Abbott *et al.*, Phys. Rev. Lett. **116**, 061102 (2016).